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The Doppler Determination of Orbits*

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The Applied Physics Laboratory has the major technical responsibility for a new satellite project which is now known as Project TRANSIT. The mission of this project is classified. However, one of the main requirements of the Project TRANSIT system is of direct interest to this meeting and is not classified. This requirement is the ability to supply to the system accurate ephemerides of special purpose satellites. The ephemerides must be supplied for short time intervals in the future, say for intervals of one-half day, and must be updated continually to provide maximum accuracy and reliability. One of the special purpose aspects of the Project TRANSIT satellites is that they shall contain whatever tracking aids that are deemed necessary to maintain the continuity and accuracy of the ephemerides.

In making feasibility studies of the Project TRANSIT system it has been necessary to make estimates of the probable accuracy with which the satellites can be tracked and the ephemerides produced. At this time, the method of tracking that shows the most promise of providing the required accuracy with the greatest economy is a radio doppler tracking system that is a natural out-growth of early doppler tracking studies performed at APL. (By the greatest economy is meant the least cost in instrumentation and operation of the ground receiving stations and the least size, weight, and cost of satellite instrumentation.) It is the purpose of this ~~talk~~ presentation to briefly summarize the results of some of the studies that have been performed at APL on the accuracy with which a satellite can be tracked with a radio doppler tracking system when the satellite is properly instrumented for such doppler tracking.

Before proceeding further, I wish to make perfectly clear what is meant in this talk by doppler tracking. It was noted earlier today that there is a dichotomy between those who prefer to use radio angular measurements of the satellite track and those who prefer to utilize the radio Doppler shift. Within the latter group there should be recognized a further subdivision into those who prefer to use only the central portion of the doppler curve to obtain the slant range and

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time of closest approach and those who prefer to utilize the whole of the doppler curve corresponding to the satellite passing from horizon to horizon. Studies of the use of the doppler shift for satellite tracking indicate that, when properly utilized, each segment of the doppler curve provides useful information about the track of the satellite. Furthermore, these studies show that in order to gain the maximum information from the doppler data, the data should be used in a direct calculation of the values of the six orbit parameters rather than used to compute intermediate parameters such as the slant range and time of closest approach. Consequently, in this talk, the term doppler tracking denotes calculations in which the orbit parameters of the satellite are calculated directly from a fit of an accurate theory of the doppler shift to all experimental doppler data--data from the "limbs" and "knees" of the doppler curves as well as the central portion of the curve near the inflection point.

Let me emphasize this approach with the following (somewhat simplified) argument. When one writes down the equations for the doppler shift as a function of the six orbit parameters and the time, neglecting refraction, experimental noise, oscillator drift, etc., one can clearly see that no two orbit parameters are "degenerate". That is, the equations cannot be factored in such a way that two of the orbit parameters collapse into one parameter for any finite interval of time. (To prove this, it is necessary to include the fact that the receiving station is rotating under the satellite orbit with the earth's rotation.) The fact that none of the orbit parameters are degenerate means that all six parameters can be determined from knowing values of the doppler shift for six time points. Consequently, the very large number of data points arising from using the whole doppler curve is roughly equivalent to providing statistical redundancy to aid in determining the six parameters when experimental noise is present. The data points of the doppler curve should then be weighted according to the characteristics of the noise that is present, and the characteristics of the noise definitely do not dictate that all but the central portion of the doppler curve should have zero weight. Of course, in practice, the problem is not quite this straightforward. Nevertheless, the principle still stands that the six orbit elements should be determined simultaneously from all doppler data and not through the intermediary of slant range determinations.

A practical demonstration of the best or ultimate accuracy to which the orbit parameters can be determined by this method cannot be given because there have not yet been satellites properly instrumented for accurate doppler tracking. Clearly, for accurate tracking the satellite transmitter must be very stable and unmodulated, and (considering weight and size limitations on American satellites) such transmitters quite

reasonably have not been placed in previous satellites. Consequently, present satellite transmissions have very poor characteristics from the standpoint of doppler tracking. Nevertheless, a very striking demonstration of the principle of using all of the doppler curve to determine simultaneously all six orbit parameters does exist. For this demonstration I shall review some of the results of the earliest doppler tracking that was performed at APL where the extreme case was taken in which only one doppler curve (single pass near a single receiving station) was used to simultaneously determine all six orbit parameters with useful accuracy even including the very poor characteristics of the satellite transmissions. (The work leading to these results are reported in detail in a Laboratory report, Bumblebee Series No. 176, "Theoretical Analysis of Doppler Radio Signals from Earth Satellites" by William H. Guier and George C. Weiffenbach. The tracking results were reported in a letter to the Editor of NATURE, Vol. 181, pg. 1525 of 31 May 1958, by the same authors.) Besides demonstrating the above principle of doppler tracking, these results indicate qualitatively the tremendous amount of orbital information that is contained in a single experimental doppler curve.

Figure I indicates the results of a determination of the orbit of Sputnik I from a single doppler curve. In determining this orbit the theoretical equations for the doppler shift were written as a function of eight independent parameters and the time. The first six of these eight parameters were the orbit elements of the satellite and the two additional parameters were included to account for incomplete knowledge of the satellite's transmitter frequency and ionospheric refraction. An accurate value of the transmitter frequency was not known and the seventh parameter was introduced as a "vernier" on the value of the transmitter frequency. The 20mc signal from Sputnik I was used for this determination and consequently the experimental doppler shift contained a large contribution from refraction. This contribution of refraction was included in the theoretical equations for the doppler shift by assuming a shape for the electron density as a function of altitude, and then the eighth parameter was introduced to represent the magnitude of the maximum of the electron density. Consequently, this orbit determination represented the simultaneous evaluation of eight parameters, not six, by performing a gradual adjustment of their values until a least squares fit of the theoretical doppler shift to the experimental data was obtained.

From Figure I it can be seen that the ground range to the satellite was relatively small--about 70 miles--and consequently the satellite passed nearly overhead. For this close pass, it can be seen that surprisingly accurate values for all six of the orbit elements were obtained. For comparison, the values of the orbit elements as determined by

Orbital Data

Sputnik I - 20 mc.

Estimated experimental error:	± 4 cps.
RMS fit to data:	± 1.6 cps.
Approximate ground range:	73 St. miles
Minimum angle of arrival from horizon:	20°
t_o = 23:47 GCT, Oct. 21, 1957	
Latitude of <u>O</u>:	38° 41' N
Longitude of <u>O</u>:	75° 59' W

Orbit Element	Doppler Determination	Ref. 4*	Ref. 5*
Period	95 min. 38 sec.	95 min. 36 sec.	95 min. 34 sec.
Eccentricity	.053	.053 ± .001	.048 ± .002
Inclination	64° 10'	64° 40' ± 10'	65°
Argument of perigee	43° 30'	-----	-----
Lat. of perigee	38° 20' N	36° ± 3° N	40.9° N
Long. of Asc. Node	289°	291.6° ± .3°	-----

* These values for the orbital elements were taken from references (4) and (5) and were given for Oct. 15. Using their values for the secular variations, the values for Oct. 15 were extrapolated to Oct. 21 for purposes of comparison.

FIGURE I

two British agencies is given. The British orbital data was believed to be the most accurate data that APL had on Sputnik I at that time, and was compiled from many sets of data including both interferometric and doppler data.

It perhaps is not too surprising that the orbit given in Figure I could be determined from a single pass since the pass was very close to the receiving station. It certainly is to be expected that the accuracy of the orbit elements would deteriorate rapidly as the minimum ground range to the satellite increases when only a single experimental doppler curve is used. Consequently, it was very surprising to find that even when the satellite pass is far from the receiving station the orbit can still be determined from a single doppler curve containing non-random errors such as refraction, frequency drift, and modulation.

Figure II indicates the results of three attempts to determine the orbit of Explorer I by this same method. From Figure II it can be seen that for these passes the minimum ground range varied from 580 to 740 miles, and the satellite was never more than about 20 degrees above the horizon. Again for comparison, at the bottom of Figure II is given the average of the three determinations of the orbit from the doppler data together with the orbit as determined by the Minitrack system.

Clearly, attempting to determine the orbit of a satellite from a single doppler curve is not the proper way to make accurate orbit determinations. These results have been presented to indicate the large amount of orbit information that is contained in the doppler shift when the data is utilized properly. Extracting this much information is not easy. Each of the four orbit determinations tabulated in Figures I and II represent about 20 hours of computing time on a Univac Scientific 1103AF computer. Since this initial work, special computing techniques have been developed to shorten the computations by better than a factor of ten. However, a delicate "touch" is still required in such orbit determinations from a single doppler curve, and for this reason no attempt has been made to make orbit determinations on a routine basis by this method.

Having established the proper approach to doppler tracking, I now wish to turn to a consideration of some aspects of a doppler tracking system that should meet the specific needs of Project TRANSIT. First of all, most of the errors that were present in the experimental data used in the orbit determinations given in Figures I and II can be eliminated when the satellites are especially designed for accurate doppler tracking. It appears perfectly feasible to construct satellite oscillators that are stable to 1 part in 10^9 during the time

(a)

Orbital Data

Explorer I - 108 mc.

	Feb. 4, 1958	Feb. 5, 1958	Feb. 6, 1958
Est. Exper. Error:	1.5 cps.	1.5 cps.	1.5 cps.
RMS fit to data:	2.1 cps.	2.5 cps.	8.4 cps.
Approx. ground range:	737 St. miles	580 St. miles	637 St. miles
Min. Angle of Arr. from horizon:	0°	0°	0°
t_o =	2:40:19 CCT	0:01:37 CCT	0:32:35 CCT
Lat. of O:	28° 33' N	38° 55' N	29° 25' N
Long. of O:	74° 54' W	78° 47' W	71° 13' W
Period:	114 min. 37 sec.	114 min. 40 sec.	114 min. 37 sec.
Eccentricity:	.134	.124	.146
Inclination:	28° 50'	32° 20'	29° 40'
Arg. of perigee:	93° 50'	113° 50'	74° 30'
Long. of Asc. Node:	40'	340° 10'	340° 40'

(b)

Comparison of Orbital Data

Orbit Element	Doppler Determination (av)	Minitrack Determination
Period:	114 min. 38 sec.	114 min. 57 sec.
Eccentricity:	.135	.141
Inclination:	30° 20'	33° 35'
Arg. of Perigee:	94°	120° 46'
Long. of Asc. Node:	347°	342° 57'

FIGURE II

of a satellite pass. This stability is several orders of magnitude better than the transmitters in Sputnik I and Explorer I. Furthermore, the present lack of knowledge of the forces acting on satellites is not a major consideration because (1) Project TRANSIT satellites can be orbited at sufficiently high altitudes that air drag is eliminated as a significant source of error over periods less than a day, and (2) with the capability of very accurate tracking, the earth's gravitational force field can be determined to the point where residual "force noise" is negligible over periods of one day. Finally, with proper receiving station instrumentation, frequency and time can be measured so accurately that instrumentation noise is certainly not a limitation on tracking accuracy. At the present time, ionospheric refraction appears to be the eventual limiting factor on accuracy. Consequently, I now wish to briefly discuss one method for sufficiently eliminating refractive effects even if the satellite transmitter frequencies are limited by practical considerations to frequencies below 500mc. Following this I will conclude with a brief summary of more recent results on the achievable accuracy of a doppler tracking system.

Figure III shows an experimental determination of the effect of ionospheric refraction on the doppler shift. This data was taken during a daytime pass of Sputnik II and the 20mc and 40mc signals were received simultaneously. Due to the frequency dependence of the index of refraction occurring in the ionosphere, the contribution of refraction to the 20mc transmission is more severe than that for the 40mc transmission. Consequently, one can see from Figure III the effect of refraction on the doppler shift is to decrease the value of the doppler shift compared to what it would be in a vacuum.

Figure IV indicates schematically the effect of refraction on the resulting orbit determination. If one attempted to determine from the doppler curve just the slant range at closest approach, the refracted doppler curve would yield too large a slant range if no correction for refractive effects were made. The same basic effect is still predominant when one determines the orbit parameters from the whole doppler curve, and in general, a refracted doppler curve tends to "push" the orbit away from the receiving station. This is indicated schematically in the bottom graph of Figure IV, where as indicated, the refractive effects produce a doppler curve that cannot be fitted exactly by any ballistic trajectory, but which, from a least squares determination, produces an orbit that is farther away from the receiving station than would otherwise be the case.

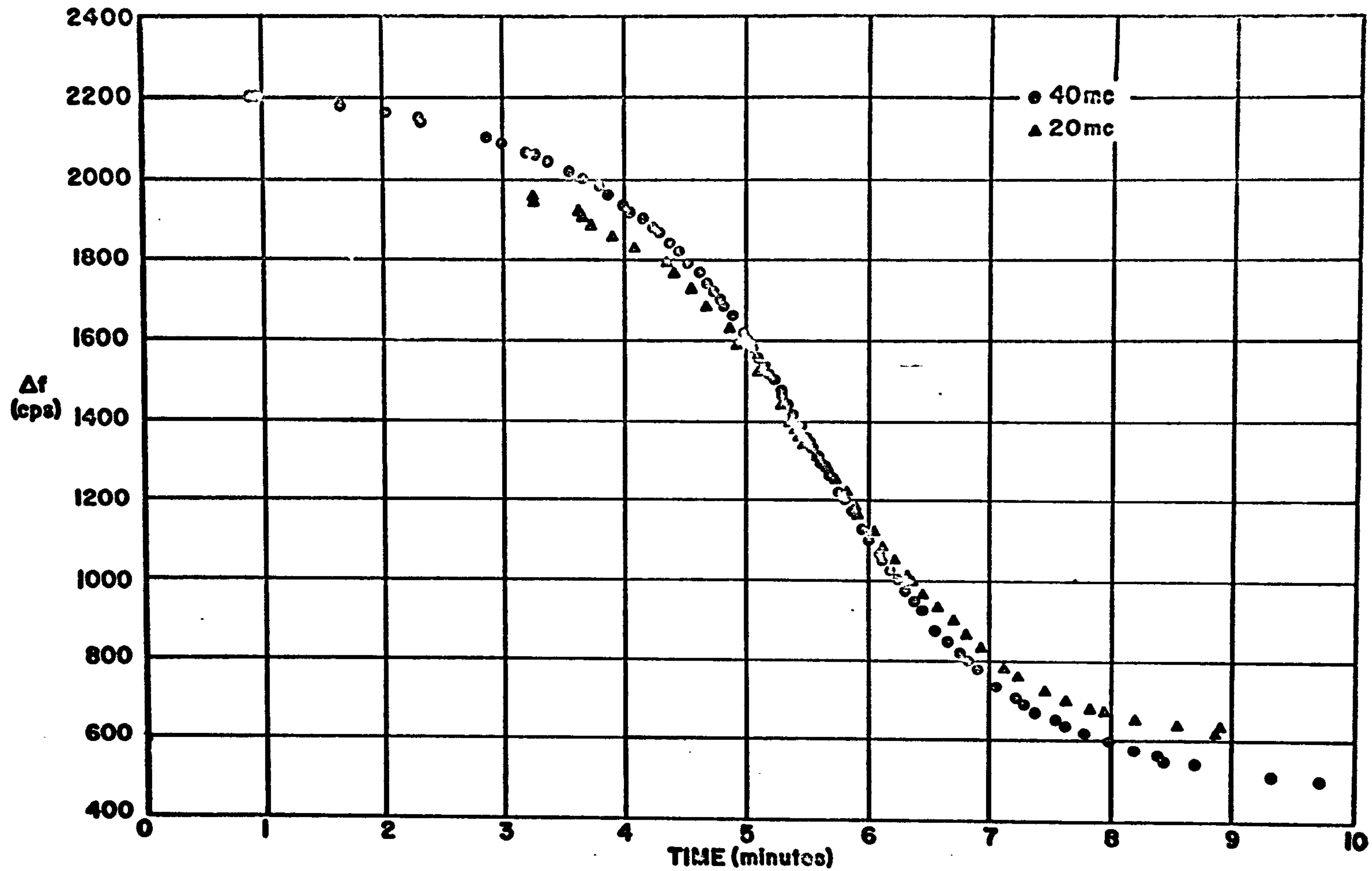
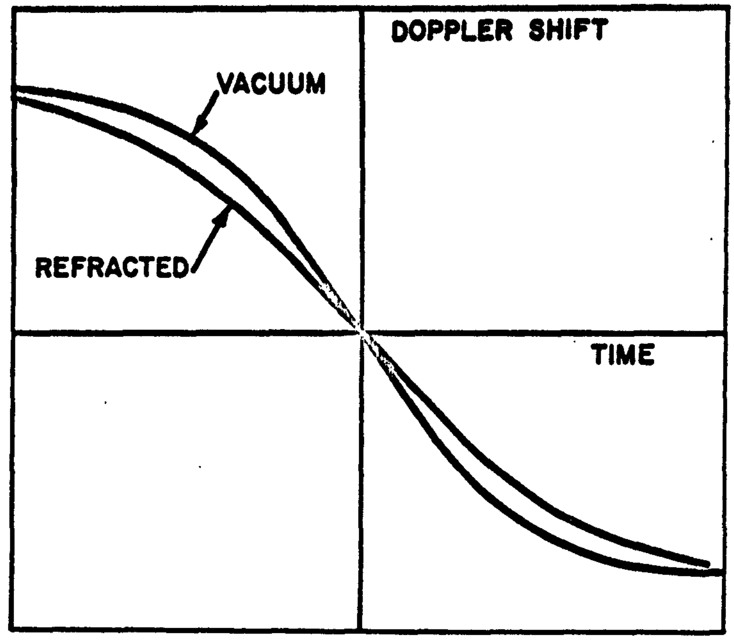


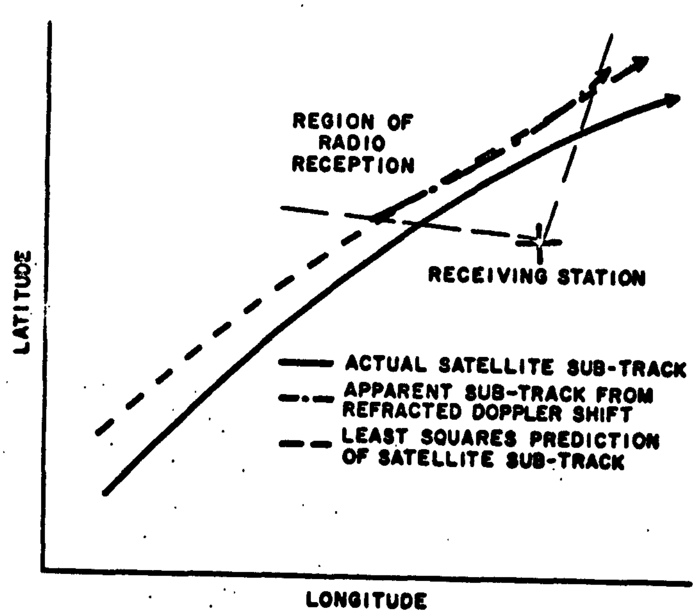
Fig.III EFFECT OF IONOSPHERIC REFRACTION ON DOPPLER SIGNAL

Experimental Observations on radio signals from Sputnik II for 11/8/57, with t = 0 at 2:15pm local time. The 20mc Doppler frequencies were doubled, and the curves adjusted to make their points of symmetry coincide. It is estimated that $f_oF_2 \sim 12$mc for this pass.

FIGURE III



SCHEMATIC OF THE IONOSPHERIC EFFECT ON DOPPLER SHIFT



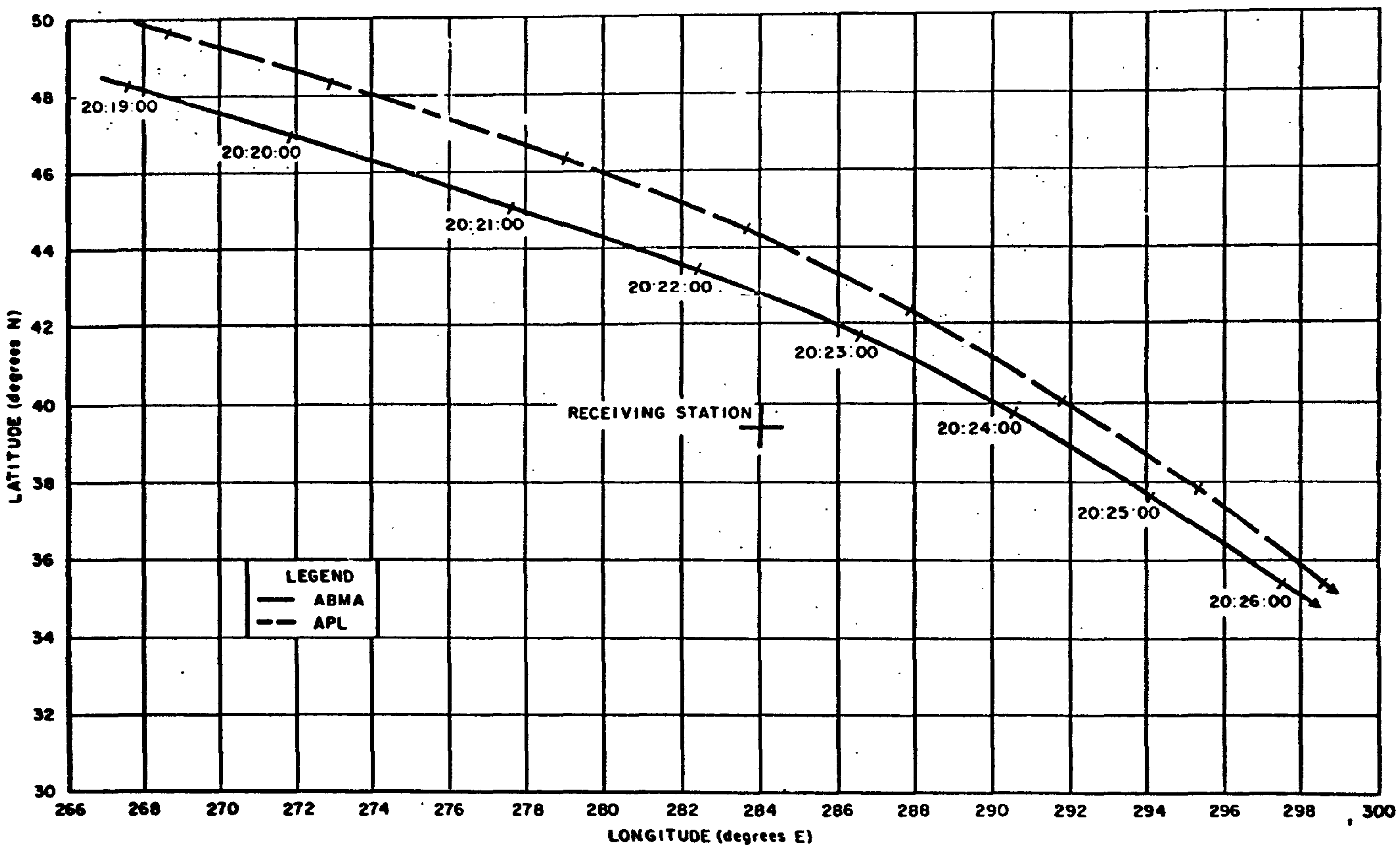
SCHEMATIC OF ERROR IN THE PREDICTED ORBIT DUE TO THE PRESENCE OF REFRACTION

FIGURE IV

Figure V indicates the results of an actual orbit determination of Explorer IV where this effect of refraction has occurred. The orbit whose subtrack is plotted in Figure V was determined in the same way as the four orbits discussed previously, except, in this case, the eighth parameter representing refractive effects was not included. Consequently, the subtrack, plotted in Figure V as a dashed line, represents the best orbit determination from a single doppler curve when the theoretical equations for the doppler shift assume the signal has traveled in a vacuum. The solid curve indicates the subtrack of Explorer IV as determined by ABMA, and it can be seen that the determination from the refracted doppler curve yielded an orbit that was "pushed" away from the receiving station. Figure VI indicates the results of a similar orbit determination of Explorer IV in which a nearly overhead pass was especially chosen. For an overhead pass the effect of refraction should push the orbit (as determined from a single doppler curve) higher in altitude and leave the subtrack relatively unchanged. It can be seen from Figure VI that the subtrack, indicated by the dot-dash curve, has not been pushed to one side of the receiving station. Also, while not indicated in this figure, the resulting altitude was too high.

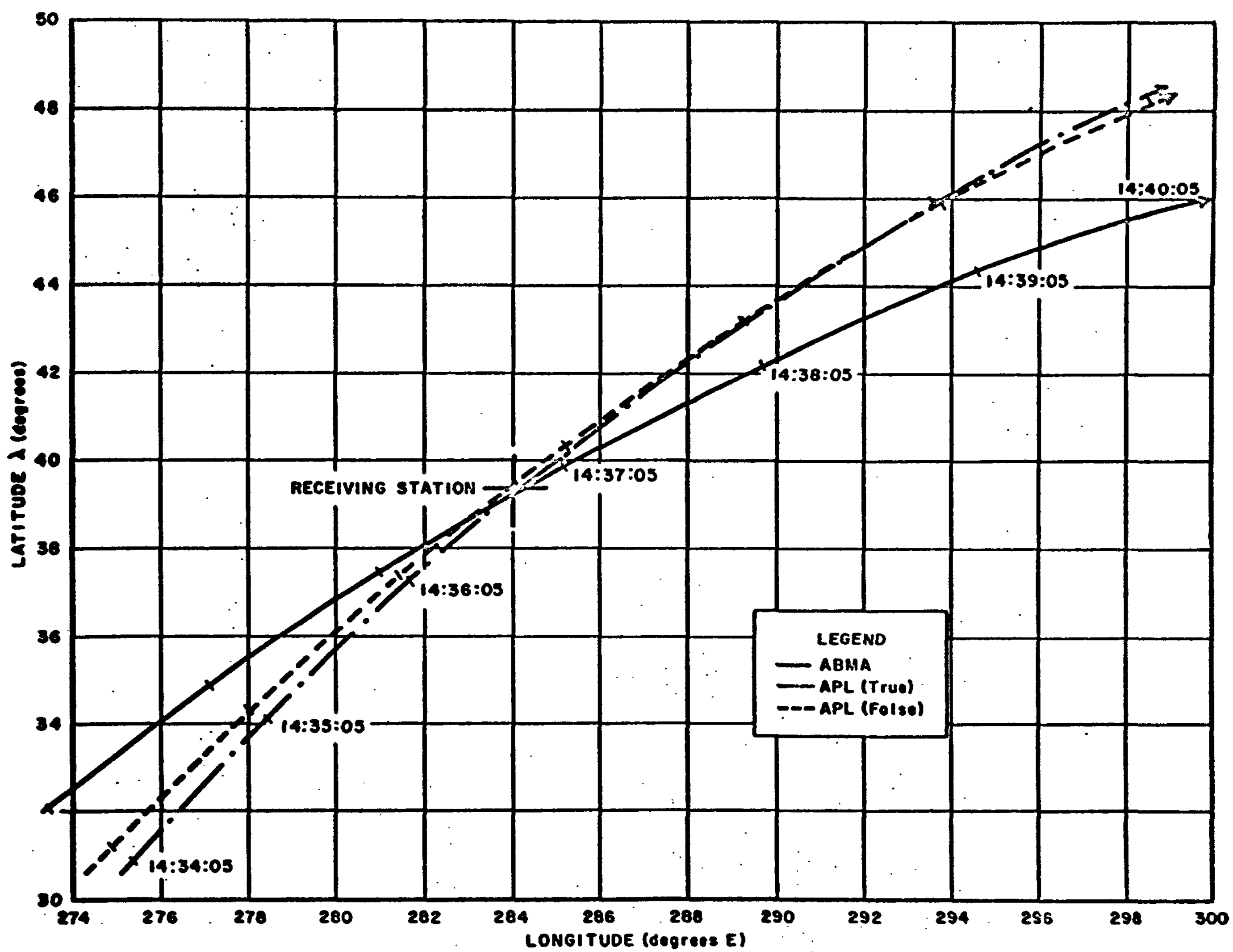
Figure VII indicates very briefly the algebraic reduction of the doppler shift to a power series in the inverse of the transmitter frequency. A good approximation to the expression for the doppler shift in the presence of a refractive medium is given by replacing the slant range from receiver to satellite by the phase integral denoted by $\Lambda(t)$ in Figure VII. Expanding the index of refraction in a power series of the inverse of the transmitter frequency, and then substituting this into the expression for the doppler shift yields an expansion indicated by Eqs. (6) in Figure VII. In Eqs. (6), the first two terms in the expansion have been explicitly given. The first term, $\Delta f_o(t)$, is the doppler shift as received in a vacuum. The second term, $\Delta f_1(t)$ represents the first order contribution of refraction to the doppler shift, and is linear in the ionosphere electron density.

Figure VIII indicates in tabular form the estimated magnitude of the various terms in the power series expansion of the refracted doppler shift. Considering the columns in the order left to right, the first is the transmitter frequency. The next column represents the estimated maximum deviation of the refractive index from unity at the various frequencies and was computed for a plasma resonance frequency of 10mc. These deviations from unity represent very roughly a one-sigma value higher than the day-to-day average that is to be expected. The third column indicates approximately the maximum value of the unrefracted doppler shift. The last



PREDICTED GROUND TRACK OF EXPLORER IV, 30 JULY 1958 (TIME IN GCT)

FIGURE V



PREDICTED GROUND TRACK OF EXPLORER IV

FIGURE VI

$$\mathcal{L}(t) = \int_{P_o(t)} dr \frac{ds_o(r)}{dr} n\left[s_o(r)\right] + O\left[(1 - n^2)^2\right] \qquad (5a)$$

Substituting Eq. (1) into Eq. (5a),

$$\mathcal{L}(t) = \int_{P_o(t)} dr \frac{ds_o}{dr} - \frac{1}{2}\int_{P_o(t)} dr \frac{ds_o}{dr}\left[1-n^2(s_o\ r)\right] + O\left[(1-n^2)^2\right] \qquad (5b)$$

or using Eq. (4):

$$\Delta f(t) = \Delta f_o(t) + \Delta f_1(t) + \text{2nd order terms} \qquad (6a)$$

where

$$\Delta f_1(t) = \frac{e^2}{2\pi m f_T c} \frac{d}{dt} \int_{P_o(t)} dr \frac{ds_o}{dr} N_e(s_o, t) \qquad (6b)$$

and where:

$s_o(r)$ = geometric path length

$P_o(t)$ = geometric transmission path from transmitter to receiver

$n(s_o)$ = index of refraction along the geometric path

$\Delta f_o(t)$ = Doppler shift in the absence of refraction

$\Delta f_1(t)$ = first order refraction correction term to Doppler shift

$N_e(s_o,t)$ = electron density along the geometric path as a function of position and time.

FIGURE VII

f	$1-n_{min}$	$\Delta f_o(t)$*	$\Delta f_1(t)$*	2nd order terms*
50mc	0.0202	1250cps	10cps	0.1cps
100mc	0.00501	2500cps	5cps	0.01cps
200mc	0.00125	5000cps	2.5cps	0.002cps
500mc	0.00020	12,500cps	1cps	-----
1000mc	0.00005	25,000cps	0.5cps	-----

*These are typical maxima of these quantities for a given Doppler curve. The numbers for the second order terms are only an order of magnitude estimate.

FIGURE VIII

two columns indicate the approximate magnitudes of the first and second order refraction effects, respectively.

From Figure VIII it can be seen that the vacuum doppler shift is directly proportional to the transmitter frequency while the first order refraction contribution is inversely proportional to the frequency. Furthermore, if a method could be found to eliminate the first order refraction contribution, the remaining refractive effects are negligible. Figure IX indicates the trivial algebra needed to eliminate the first order refraction effect if one has available two simultaneous experimental doppler curves for two frequencies, for example, 100mc and 200mc, and thereby obtain an extrapolation to a vacuum doppler curve. This extrapolated vacuum doppler curve can then be used for tracking computations with very little residual effect of the ionosphere present. Of indirect interest, Eq. (9) of Figure IX indicates the result if the vacuum doppler shift is algebraically eliminated and the first order refraction contribution itself is computed. This equation, considered as a function of time, may then be used to attempt measurements of the electron density itself in the ionosphere.

At the present time it appears that practical considerations will limit the transmitter frequencies in the Project TRANSIT satellites to values too small to allow direct neglect of the effects of the ionosphere (radar frequencies). If this limitation remains, the effect of the ionosphere will be treated by the method just discussed. From studies such as reported here it now appears that this method will eliminate the effects of refraction to the point where their contribution to the doppler shift no longer represents a major bias in the doppler tracking data. In fact, the remaining contributions have more nearly the characteristics of random noise than biases. Consequently, assuming that this dual frequency method presents no refraction biases in the experimental data, assuming that the satellite transmitters can be made sufficiently stable, and finally assuming that eventually the earth's gravitational field is determined, there remains only the effect of random errors to contribute to the errors in the Project TRANSIT orbit determinations.

Let me now summarize the results of those studies that have been made concerning the probable error in tracking of the satellites that results from random errors. The characteristics of these errors arising from instrumentation noise, residual refractive effects, etc., have been estimated to be below a one-sigma value of 0.5 cps. at transmitter frequencies of 200mc with a correlation time that is not well known, but certainly less than ten seconds. In making error analyses when random noise is present, one can use the standard techniques

Let a satellite contain two stable transmitters with frequencies f_1 and f_2. The Doppler shifts can be written to first order as

$$\Delta f^{(1)}(t) = \Delta f_o^{(1)}(t) + \Delta f_1^{(1)}(t)$$

$$\begin{aligned} \Delta f^{(2)}(t) &= \Delta f_o^{(2)}(t) + \Delta f_1^{(2)}(t) \\ &= \frac{f_2}{f_1} \Delta f_o^{(1)}(t) + \frac{f_1}{f_2} \Delta f_1^{(1)}(t) \end{aligned} \tag{7}$$

Eliminating $\Delta f_1^{(1)}(t)$ from these equations,

$$\frac{1}{c} \frac{d}{dt} \Lambda_o(t) = \frac{f_2 \; \Delta f^{(2)}(t) - f_1 \Delta f^{(1)}(t)}{f_2^{\;2} - f_1^{\;2}} \tag{8}$$

where $\frac{d}{dt} \Lambda_o(t)$ is the time rate of change of the geometric path length from transmitter to receiver, i.e., the effects of refraction have been eliminated to first order, so that, to this accuracy, an "unrefracted" Doppler shift is available for orbit determination.

Similarly, by eliminating Δf_o from Eqs. (7), and using Eqs. (6), the following relationship is obtained:

$$\frac{e^2}{2\pi mc} \frac{d}{dt} \int_{P_o(t)} dr \frac{ds_o}{dr} N_e(s_o, t) = \frac{f_1^{\;2} f_2 \Delta f^{(2)}(t) - f_2^{\;2} f_1 \Delta f^{(1)}(t)}{f_1^{\;2} - f_2^{\;2}} \tag{9}$$

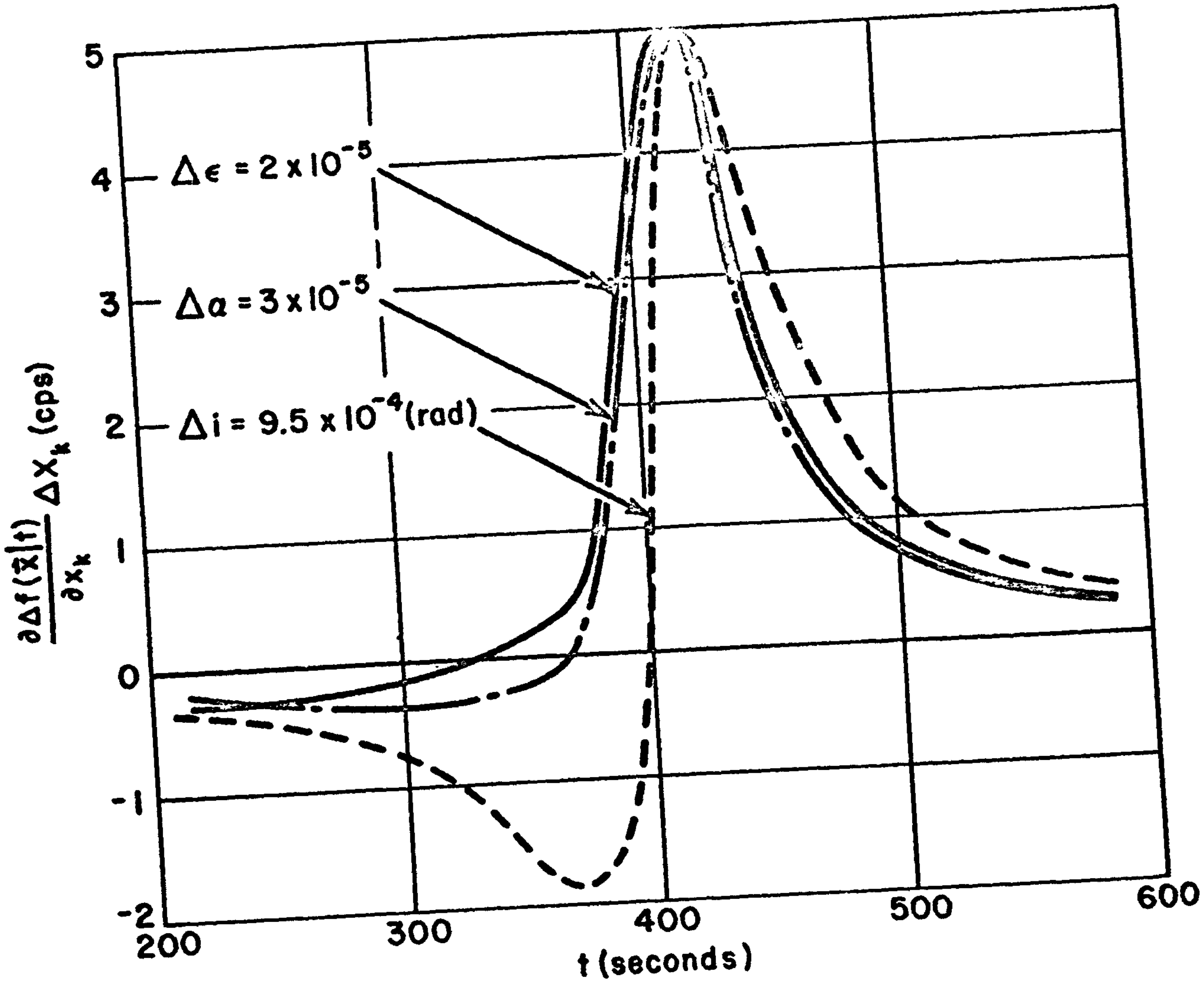
which indicates how one might experimentally study the distribution of free electrons in the ionosphere per se.

FIGURE IX

of linearizing the equations about the correct values, and then computing the expected error contributed by the noise by taking ensemble averages over all possible noise samples. However, I shall take a different approach in this talk in order to present the results in a way that gives a better intuitive feel of the expected error. Suffice to say that more erudite error studies indicate that when predictions of the satellite are not attempted for longer periods than about one-half day, when there are sufficient tracking stations that about two experimental doppler curves are obtained per satellite revolution, and when only random errors of less than 0.5 cps. RMS out of 200mc are present, the satellite orbit should be predictable in position to better than 0.1 miles RMS.

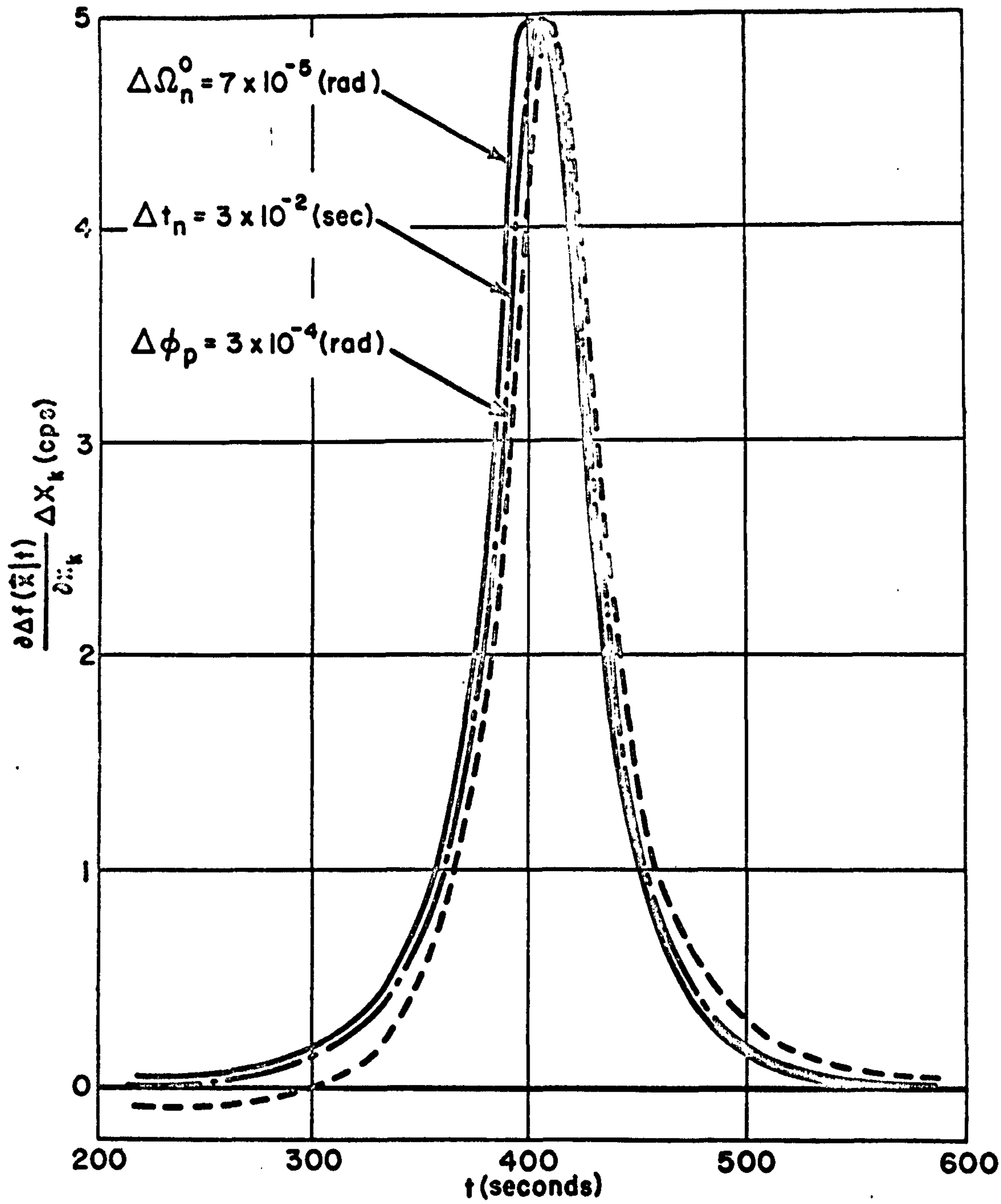
A feel for how well the orbit elements of the satellite can be determined from the doppler data can be obtained by examining the change in the doppler shift with a small change in each of the orbit elements. Figure X shows an example of the way the doppler shift is changed with changes in the eccentricity, semi-major axis (scaled by the earth's radius) and the inclination. In Figure X the changes in the orbit elements have been varied to normalize each of the curves so that they look as closely alike as possible. Remembering that the experimental data should be good to less than a half cycle random error, it can be seen that the three curves are sufficiently different to enable a determination of all three simultaneously from a single doppler curve to about the error indicated in Figure X. On the other hand, Figure XI indicates how the remaining three orbit elements affect the doppler curve, and for this case a simultaneous determination of these three orbit elements from a single doppler curve would most likely produce much larger errors. The detailed shape of these curves change markedly with varying geometry of the pass relative to the receiving station so that for one given pass there will be large sensitivity to some of the orbit elements, while for a different pass other orbit elements will exhibit the large sensitivity. Therefore, taking a collection of different passes, as would naturally be done in actual tracking, the orbit elements can be determined quite accurately.

Another way to examine the probable error that will result from doppler tracking is to examine the shape of the mean square difference between the theoretical and experimental doppler shift as a function of the six orbit elements. In Figures XII through XV are shown typical traces through the mean square fit of the data to the theory for random errors of about 0.5 cps. RMS. The ordinate in each graph indicates the mean square fit in $(cps)^2$ and the two abcissae in each figure indicate changes in the orbit elements as well as the error in miles that result from such a change in each



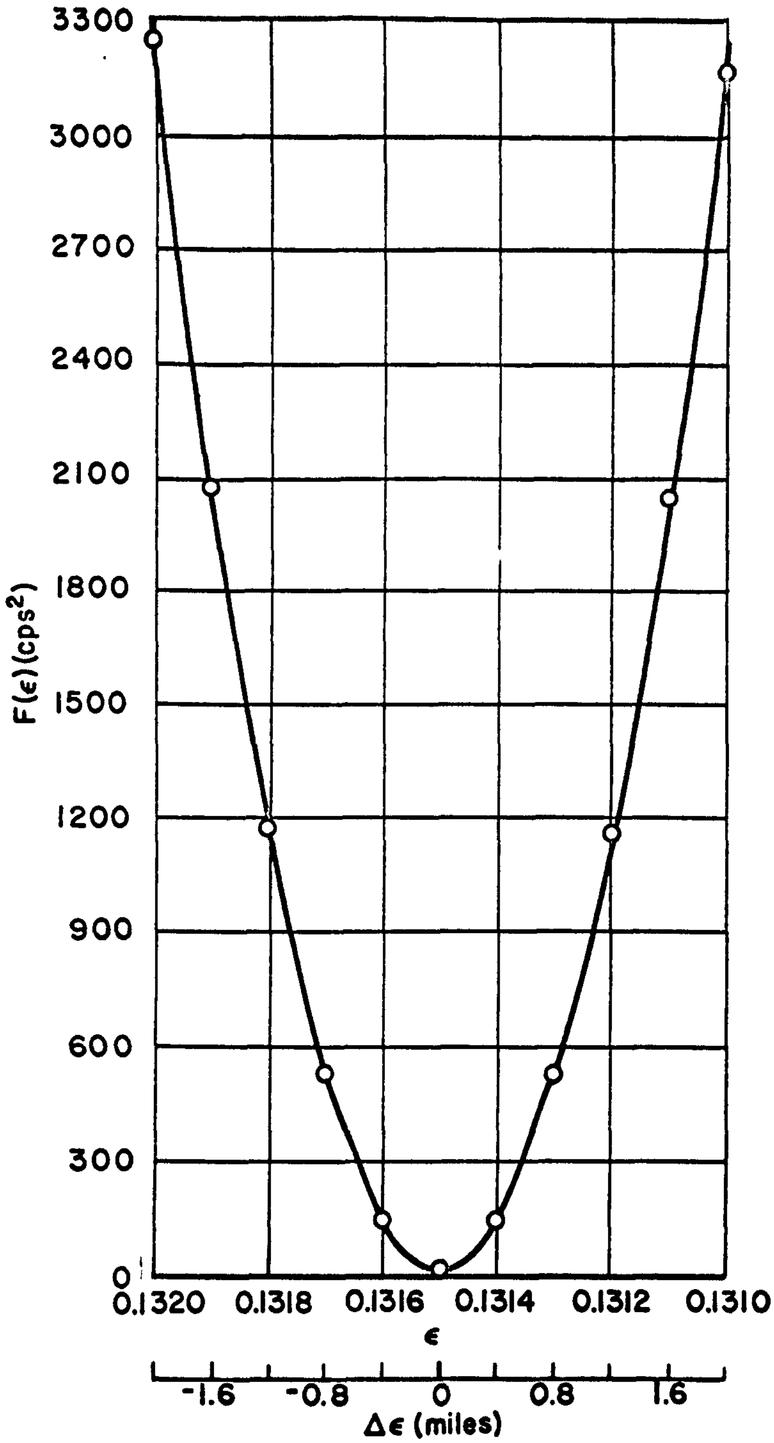
DEPENDENCE OF DOPPLER SIGNAL ON ORBIT ELEMENTS

FIGURE X



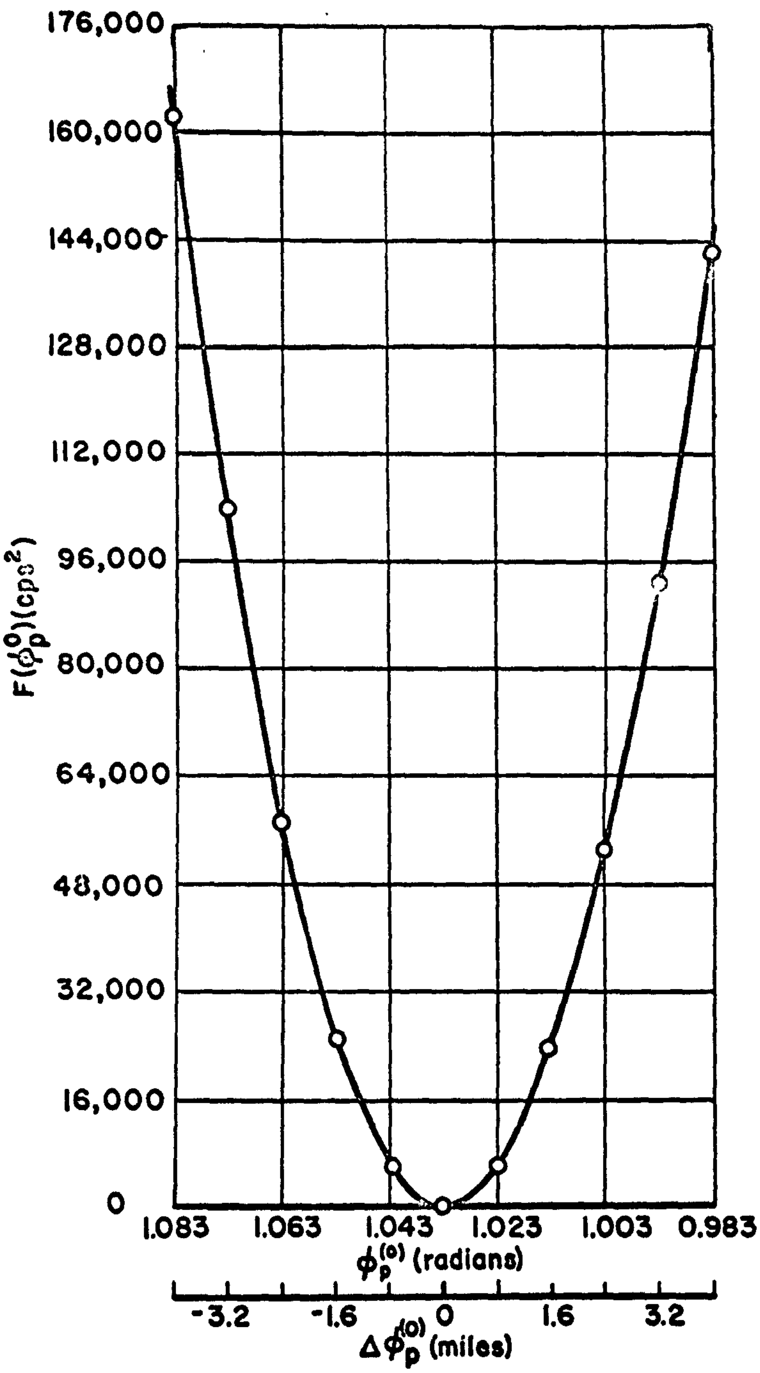
DEPENDENCE OF DOPPLER SIGNAL ON ORBIT ELEMENTS

FIGURE XI



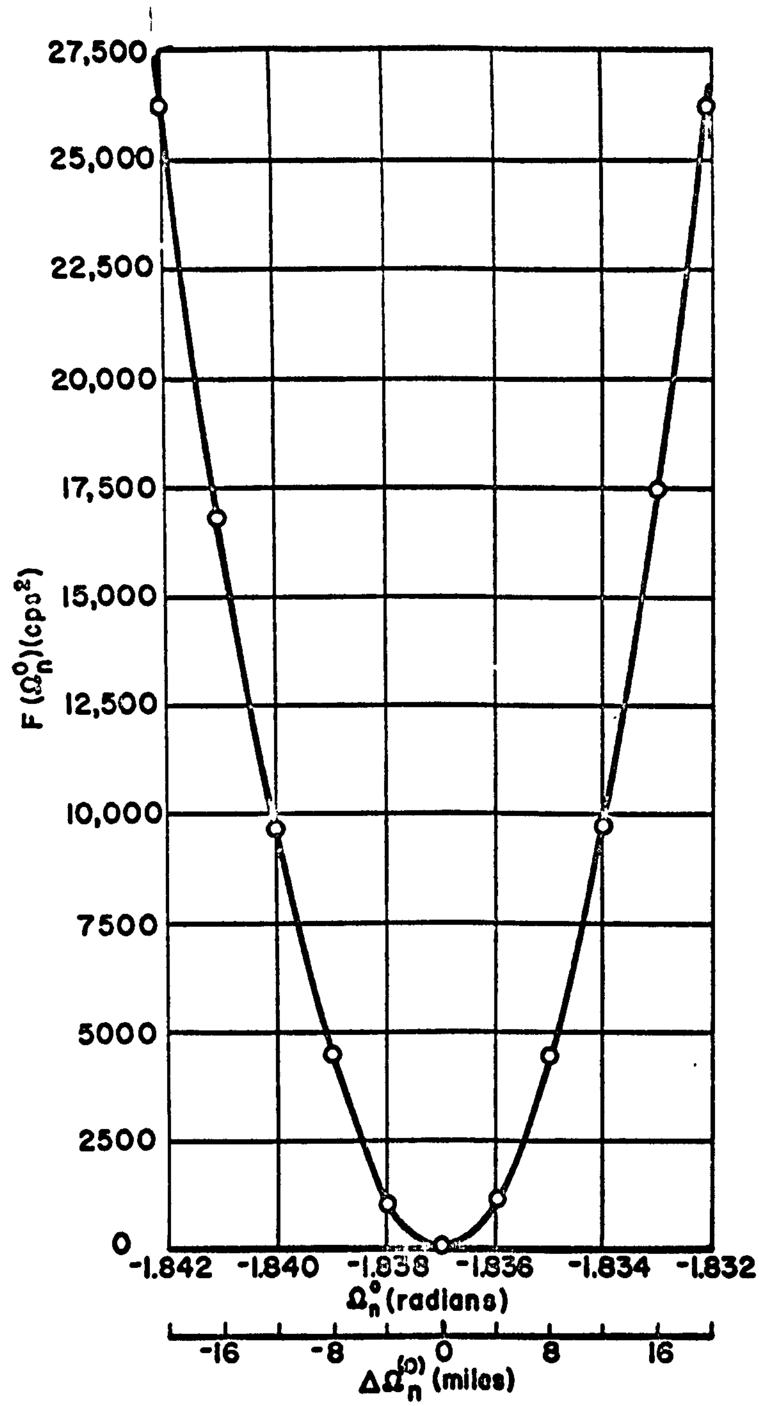
ECCENTRICITY F(E) VERSUS E

FIGURE XII



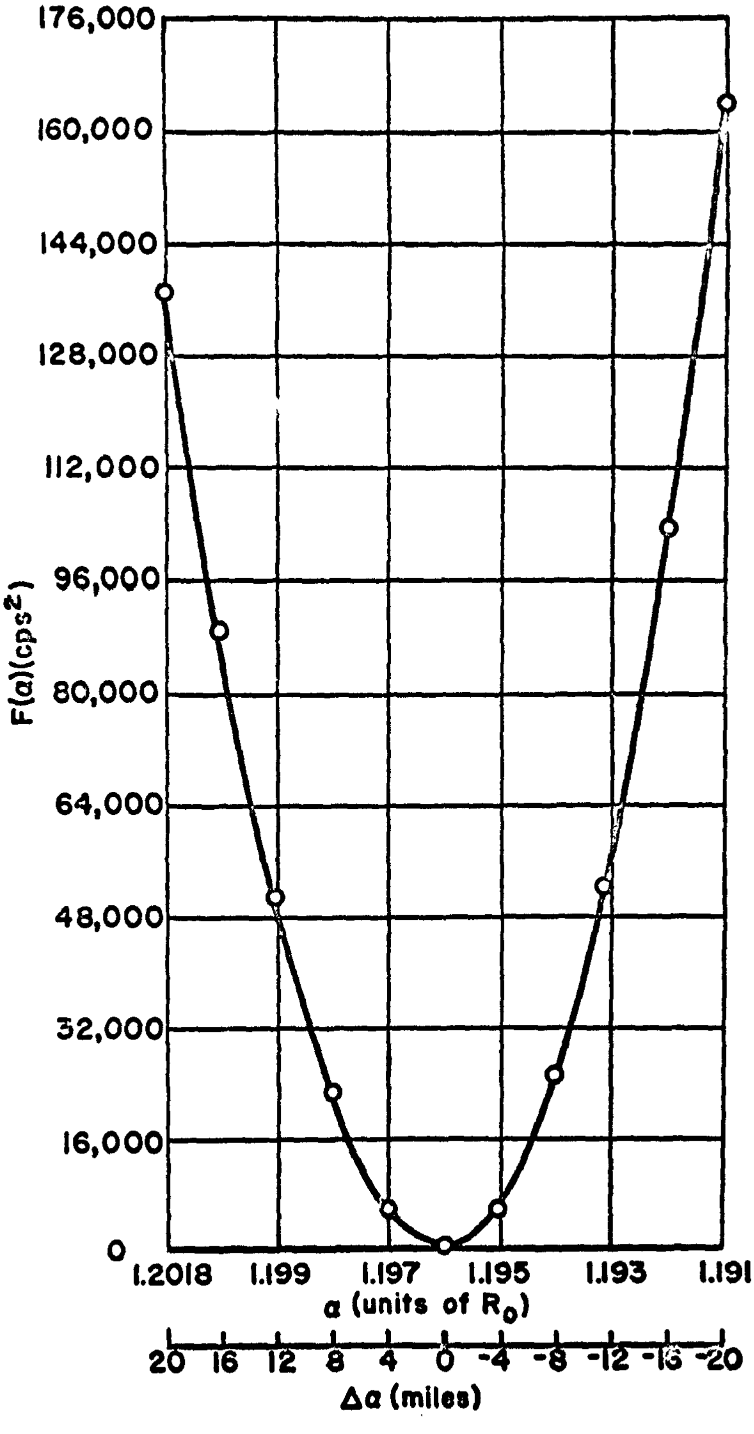
ARGUMENT OF PERIGEE $F(\phi_p^{(o)})$ VERSUS $\phi_p^{(o)}$

FIGURE XIII



LONGITUDE OF ASCENDING NODE $F(\Omega_n^{(o)})$ VERSUS $\Omega_n^{(o)}$

FIGURE XIV



SEMIMAJOR AXIS

FIGURE XV